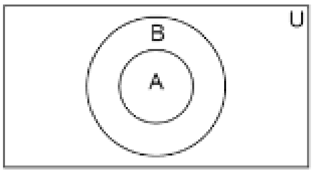
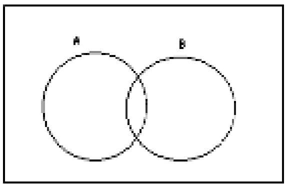
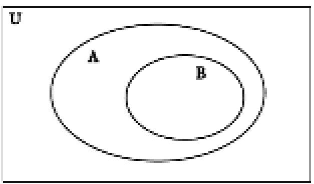
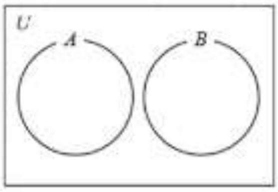
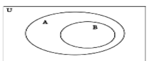
B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

MID APRIL TEST (2026-27)

Marking Scheme Class XII

Mathematics (Code – 041)

1.	If $f(x) = \|x\| + \|x-1\|$, then which of the following is correct? a. $f(x)$ is both continuous and differentiable at $x = 0$ and $x = 1$. b. $f(x)$ is both differentiable but not continues $x = 0$ and $x = 1$. c. $f(x)$ is continues but not differentiable at $x = 0$ and $x = 1$ d. $f(x)$ is neither continues and differentiable at $x = 0$ and $x = 1$.	
Answer	**c. $f(x)$ is continues but not differentiable at $x = 0$ and $x = 1$**	**1**
2.	The function $f(x) = x^2 - 4x + 6$ is increasing in the interval a. $(0, 2)$ c. $[1,2]$ b. $(-\infty, 2]$ d. $[2, \infty)$	
Answer	**d.** $[2, \infty)$	**1**
3.	If A denotes the set of continuous functions and B denotes set of differentiable functions then which of the following depicts the correct relation between set A and set B . a.  c.  b.  d. 	
Answer	**b.** 	**1**

4.	If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$ then the value of a is : **a.** 1 c. ± 1 **b.** -1 d. 0	
Answer	**c.** ± 1	**1**
5.	Let $f(x) = max\{(1-x), (1+x), 2\}$ number of points where f is not differentiable 1 a. 0 b. 2 b. 1 d. 3	
Answer	**b. 2**	**1**
6.	**Questions number 6** are Assertion and Reason based question. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes $(A), (B), (C)$ *and* (D) as given below a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). b. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). c. Assertion (A) is true, but Reason (R) is false. d. Assertion (A) is false, but Reason (R) is true. **Assertion** $(\boldsymbol{A})$: $f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ **Reason**$(\boldsymbol{R})$ **:** When $x \to 0$, $\sin\frac{1}{x}$ is a finite value between -1 and 1 .	
Answer	**a.** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**

SECTION – {B}

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each)

7.	a. Differentiate $2^{\cos^2 x}$ $w.r.t$ $\cos^2 x$	
Answer	$\frac{d}{d(\cos^2 x)}\left(2^{\cos^2 x}\right) = 2^{\cos^2 x} \ln 2$	**1**

	$$\mathbf{2^{\cos^2 x}\ln 2}$$	**1**
	OR b. If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$	
Answer	$$\ln x = \frac{x}{y}$$ $$\frac{1}{x} = \frac{y\cdot 1 - x\frac{dy}{dx}}{y^2}$$ $$\frac{1}{x} = \frac{y - x\frac{dy}{dx}}{y^2}$$ $$y^2 = x\left(y - x\frac{dy}{dx}\right)$$ $$y^2 = xy - x^2\frac{dy}{dx}$$ $$x^2\frac{dy}{dx} = xy - y^2$$ $$x^2\frac{dy}{dx} = y(x-y)$$ $$\frac{dy}{dx} = \frac{y(x-y)}{x^2}$$ $$\frac{dy}{dx} = \frac{x-y}{x\ln x}$$ $$\boxed{\frac{dy}{dx} = \frac{x-y}{x\log x}}$$	½ 1 ½ ½
8.	Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x where $x \epsilon (0,1)$	
Answer	$$x = \tan\theta,$$ $$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$ $$= \cos^{-1}(\cos 2\theta) = 2\theta,$$ $$\theta = \tan^{-1} x \Rightarrow$$ $$y = 2\tan^{-1} x,$$ $$\frac{dy}{dx} = \frac{2}{1+x^2}$$	½ ½ ½ ½

SECTION – {C}

(This section comprises of short answer type questions (SA) of 3 marks each)

9.	A. Find k so that $$f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x + 1}, & x \neq -1 \\ k\,, & x = 1 \end{cases}$$ Is continuous at $x = -1$.	
Answer	$Given\ f(x)\ is\ continuous\ at\ x = -1$	
	$\therefore f(-1) = \lim_{x \to -1} f(x)$	**½**
	$\therefore k = \lim_{x \to -1} \dfrac{x^2 - 2x - 3}{x + 1}$	**1**
	$= \lim_{x \to -1} \dfrac{(x - 3)(x + 1)}{x + 1}$	**1**
	$= \lim_{x \to -1} (x - 3)$	
	$= -1 - 3 = -4$	
	$\therefore k = -4$	**½**
	OR	
	Check the differentiability of function $f(x) = x\|x\|$ at $x = 0$.	

Answer	**Differentiability at x = 0**: $L f'(0) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h}$ $= \lim_{h\to 0^-} \frac{-h^2-0}{h}$ $= \lim_{h\to 0^-} (-h)$ ...[∵ $h \to 0$, ∵ $h \neq 0$] $= 0$ $R f'(0) = \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h}$ $= \lim_{h\to 0^+} \frac{h^2-0}{h}$ $= \lim_{h\to 0^+} (h)$ ...[∵ $h \to 0$, ∵ $h \neq 0$] $= 0$ ∴ $L f'(0) = R f'(0)$ ∴ f(x) is differentiable at x = 0.	½ 1 ½ 1
10.	Find the intervals in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is i. Increasing ii. Decreasing	
Answer	When $(x-a)(x-b) > 0$ with $a < b, x < a$ or $x > b$. When $(x-a)(x-b) < 0$ with $a < b, a < x < b$. $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$ $f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$ $= \frac{15}{2}x^{\frac{1}{2}}(1-x)$ Here , 0, 1 are the roots . The possible intervals are $(-\infty, 0), (0,1)$ and $(1, \infty)$ ...(1) For f(x) to be increasing, we must have $f'(x) > 0$	1 ½

	$\Rightarrow \frac{15}{2} x^{\frac{1}{2}} (1-x) > 0$	
	$\Rightarrow x \in (0,1)$	½
	So, f(x) is increasing on $(0,1)$.	
		
	For f(x) to be decreasing, we must have	
	$f'(x) < 0$	
	$\Rightarrow \frac{15}{2} x^{\frac{1}{2}} (1-x) < 0$	½
	$\Rightarrow x \in (1,\infty)$	
	So, f(x) is decreasing on $(1,\infty)$.	½
		

SECTION – {D}

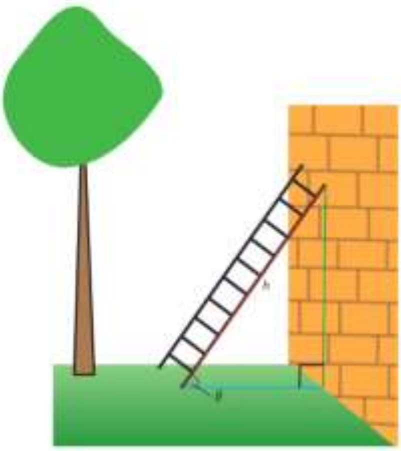
(This section comprises of long answer-type questions (LA) of 5 marks each)

11.	$$y\sqrt{(x^2+1)} = log\left(\sqrt{(x^2+1)} - x\right)$$ Then show that $$(x^2+1)dy/dx + xy + 1 = 0$$	

Answer	We have, $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$	
	Differentiating with respect to x, we get,	
	$\Rightarrow \frac{d}{dx}\left(y\sqrt{x^2+1}\right) = \frac{d}{dx}\log\left(\sqrt{x^2+1} - x\right)$ [using product rule and chain rule]	½
	$\Rightarrow y\frac{d}{dx}\left(\sqrt{x^2+1}\right) + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)} \times \frac{d}{dx}\left(\sqrt{x^2+1} - x\right)$	
	$\Rightarrow \frac{y}{2\sqrt{x^2+1}} \times \frac{d}{dx}(x^2+1) + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)} \times \left[\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}(x^2+1) - 1\right]$	½
	$\Rightarrow \frac{2xy}{2\sqrt{x^2+1}} + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1}-x\right)}\left[\frac{2x}{2\sqrt{x^2+1}} - 1\right]$	1
	$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx} = \left[\frac{1}{\sqrt{x^2+1}-x}\right]\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right] - \frac{xy}{\sqrt{x^2+1}}$	
	$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}} - \frac{xy}{\sqrt{x^2+1}}$	1
	$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx} = \frac{-(1+xy)}{\sqrt{x^2+1}}$	1
	$\Rightarrow \left(x^2+1\right)\frac{dy}{dx} = -(1+xy)$	
	$\Rightarrow \left(x^2+1\right)\frac{dy}{dx} + 1 + xy = 0$	1
	OR	
	Find differential of $\left[x^{(cotx)} + (2x^2-3)/(2x^2-x+2)\right]$ with respect to x .	
	Then $\log y = \log\left[\frac{(x^2+2x+2)^{\frac{3}{2}}}{(\sqrt{x}+3)^3(\cos x)^x}\right]$	1
	$= \log(x^2+2x+2)^{\frac{3}{2}} - \log(\sqrt{x}+3)^3(\cos x)^x$	
	$= \frac{3}{2}\log(x^2+2x+2) - 3\log(\sqrt{x}+3) - x\log(\cos x)$	½
	Differentiating both sides w.r.t. x, we get	
	$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{3}{2}\frac{d}{dx}\left[\log(x^2+2x+2)\right] - 3\frac{d}{dx}\left[\log(\sqrt{x}+3)\right] - \frac{d}{dx}\left[x\log(\cos x)\right]$	
	$= \frac{3}{2} \times \frac{1}{x^2+2x+2}\cdot\frac{d}{dx}(x^2+2x+2) - 3 \times \frac{1}{\sqrt{x}+3}\cdot\frac{d}{dx}(\sqrt{x}+3) - \left\{x\frac{d}{dx}[\log(\cos x)] + \log(\cos x)\cdot\frac{d}{dx}(x)\right\}$	1
	$= \frac{3}{2(x^2+2x+2)} \times (2x+2\times 1+0) - \frac{3}{\sqrt{x}+3} \times \left(\frac{1}{2\sqrt{x}}+0\right) - \left\{x \times \frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x) + \log(\cos x) \times 1\right\}$	1
	$= \frac{3(2x+2)}{2(x^2+2x+2)} - \frac{3}{2\sqrt{x}(\sqrt{x}+3)} - \left\{x \times \frac{1}{\cos x}\cdot(-\sin x) + \log(\cos x)\right\}$	1
	$\therefore \frac{dy}{dx} = y\left[\frac{3(x+1)}{x^2+2x+2} - \frac{3}{2\sqrt{x}(\sqrt{x}+3)} + x\tan x - \log(\cos x)\right]$	
	$= \frac{(x^2+2x+2)^{\frac{3}{2}}}{(\sqrt{x}+3)^3(\cos x)^x}\left[\frac{3(x+1)}{x^2+2x+2} - \frac{3}{2\sqrt{x}(\sqrt{x}+3)} + x\tan x - \log(\cos x)\right]$.	½

SECTION – {E}

(This section comprises of case-study/passage-based question of 4 marks)



A ladder of fixed length ' h ' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

i. Express the distance (y) between the wall and foot of the ladder in terms of ' h ' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer.

ii. Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point.

1. Expressing y in terms of h and x:

From the Pythagorean theorem, we have:

$$h^2 = x^2 + y^2$$

Rearranging gives:

$$y^2 = h^2 - x^2$$ **1**

Taking the square root:

$$y = \sqrt{h^2 - x^2}$$

	2. **Finding the area (A) of the triangle:** The area of a triangle is given by: $$A = \frac{1}{2} \times \text{base} \times \text{height}$$ Here, the base is 'y' and the height is 'x', so: $$A = \frac{1}{2} \times y \times x = \frac{1}{2} \times \sqrt{h^2 - x^2} \times x$$ $A = \frac{1}{2}x\sqrt{h^2 - x^2}$	1
	Using the product rule: $$\frac{dA}{dx} = \frac{1}{2}\left(\sqrt{h^2 - x^2} + x \cdot \frac{d}{dx}(\sqrt{h^2 - x^2})\right)$$	½
	The derivative of $\sqrt{h^2 - x^2}$ is: $$\frac{d}{dx}(\sqrt{h^2 - x^2}) = \frac{-x}{\sqrt{h^2 - x^2}}$$	½
	Therefore: $$\frac{dA}{dx} = \frac{1}{2}\left(\sqrt{h^2 - x^2} - \frac{x^2}{\sqrt{h^2 - x^2}}\right)$$ Simplifying: $$\frac{dA}{dx} = \frac{1}{2} \cdot \frac{h^2 - 2x^2}{\sqrt{h^2 - x^2}}$$	
	2. **Finding critical points:** Set $\frac{dA}{dx} = 0$: $$h^2 - 2x^2 = 0 \Rightarrow x^2 = \frac{h^2}{2} \Rightarrow x = \frac{h}{\sqrt{2}}$$	1